FINAL EXAM MATH1680 Su/2018

Stuart Jones

Directions: *Below are 33 questions, each worth 3 points each (plus 1 question worth 1 point if you write your name).*

You can come see me in my office when the Fall semester starts back to obtain your Final Exam.

Final Exam Grade: ____ Mean Average: ____ Median Average: ____
FINAL GRADE: ____

Question	Points	Score
1	1	
2	3	
3	3	
4	3	
5	3	
6	3	
7	3	
8	3	
9	3	
10	3	
11	3	
12	3	
13	3	
14	3	
15	3	
16	3	
17	3	
18	3	
19	3	
20	3	
21	3	
22	3	
23	3	
24	3	
25	3	
26	3	
27	3	
28	3	
29	3	
30	3	
31	3	
32	3	
33	3	
34	3	
Total:	100	

1. (1 point) There is no question here. This point is awarded simply for writing your name at the top of this paper.

2. (3 points) Evaluate

$$\lim_{x \to 1} \frac{4x^2 - 3x - 1}{x^2 - 1}$$

   A. 4
   B. $\frac{0}{0}$
   C. -4
   D. $\frac{5}{2}$
   E. Does Not Exist

3. (3 points) Evaluate

$$\lim_{x \to -\infty} \frac{4x^5 - 7x + 3}{2x^5 - 2x - 1}$$

   A. 2
   B. -2
   C. 0
   D. ∞
   E. $-\infty$

4. (3 points) Evaluate

$$\lim_{x\to 1} \begin{cases} \ln x & x \leq 1 \\ \sqrt{x} - 1 & x > 1 \end{cases}$$

A. 1

B. 0

C. Does Not Exist because left-sided limit is undefined (natural log can't be negative)

D. Does Not Exist because right-sided limit is undefined (square root can't be negative)

E. Does Not Exist because the left and right one-sided limits are not the same

5. (3 points) Where, if anywhere, is $f(x)$ not continuous?

$$f(x) = \begin{cases} \frac{2x^2-10x-28}{x^2-2x-35} & x \neq 7, x \neq -5 \\ 0 & x = -5 \\ \frac{3}{2} & x = 7 \end{cases}$$

A. x=7 only

B. x=-5 only

C. x=7 and x=-5

D. x=7, x=-5, and x=5

E. $f(x)$ is continuous everywhere

6. (3 points) Find the derivative of $f(x) = \frac{2}{x-1}$, using the 4-step defintion.

Solution:

$$\text{Step 1: } f(x+h) \implies \frac{2}{x+h-1}$$

$$\text{Step 2: } f(x+h) - f(x) \implies \frac{2}{x+h-1} - \frac{2}{x-1} = \frac{2(x-1)}{(x-1)(x+h-1)} - \frac{2(x+h-1)}{(x-1)(x+h-1)} =$$

$$\frac{2x-2-2x-2h+2}{(x-1)(x+h-1)} = \frac{-2h}{(x-1)(x+h-1)}$$

$$\text{Step 3: } \frac{f(x+h)-f(x)}{h} \implies \frac{\frac{-2h}{(x-1)(x+h-1)}}{h} = \frac{-2h}{(x-1)(x+h-1)} \cdot \frac{1}{h} = \frac{-2}{(x-1)(x+h-1)}$$

$$\text{Step 4: } \lim_{h \to 0} \frac{f(x+h)-f(x)}{h} \implies \frac{-2}{(x-1)(x+0-1)} = \frac{-2}{(x-1)(x-1)} = \frac{-2}{(x-1)^2}$$

7. (3 points) Find the equation of the tangent line of $f(x) = x^2(x-3)^3$ at the point $(1, 0)$.

   A. $y = -4x + 4$

   B. $y = -4x$

   C. $y = 4x$

   D. $y = 4x + 8$

   E. None of the Above

8. (3 points) Find the first derivative. $f(x) = [(x-2)^2(x-1)]^3$

   A. $3[(x-2)^2(x-1)]^2$

   B. $6(x-2)^5(x-1)^3 + 3(x-2)^6(x-1)^2$

   C. $3[(2(x-2)(x-1)^3 + 3(x-2)^2(x-1)^2]^2$

   D. $5(x-3)^4$

   E. $3[(x-2)^2(x-1)]^2 6(x-2)(x-1)^2$

9. (3 points) Find the first derivative. $f(x) = \frac{\sqrt[3]{2x^2-1}}{\sqrt{(x-2)(4x+5)}}$

   A. $\sqrt[5]{\frac{(4x)(x-2)(4x+5)-(2x^2-1)(8x-3)}{((x-2)(4x+5))^2}}^{4}$

   B. $\frac{\sqrt[3]{2x^2-1}}{\sqrt{(x-2)(4x+5)}}[3\frac{4x}{(2x^2-1)} - 2\frac{8x-3}{(x-2)(4x+5)}]$

   C. $\frac{\frac{2x^2-1}{3\sqrt[3]{2x^2-1}}}{\frac{(x-2)(4x+5)}{2\sqrt{(x-2)(4x+5)}}}$

   D. $\frac{\sqrt[3]{2x^2-1}}{\sqrt{(x-2)(4x+5)}}[\frac{4x}{3(2x^2-1)} - \frac{1}{2}(\frac{1}{x-2} + \frac{4}{4x+5})]$

   E. The limit does not exist anywhere, so this derivative is undefined.

10. (3 points) Find the first derivative. $f(x) = \frac{x-4}{x^2+6x-4}$

    A. $\frac{1}{2x+6}$

    B. $\frac{x(x^2+6x-4)-(x-4)(2x+6)}{(x^2+6x-4)^2}$

    C. $3x + 4$

    D. $x^2 +^x -4 + (x-4)(2x+6)$

    E. $\frac{x^2+6x-4-(x-4)(2x+6)}{(x^2+6x-4)^2}$

11. (3 points) Suppose a phone company is selling a particular model of phone. Their research finds a monthly cost function of $C(x) = 200x + 15,000$ and a monthly demand function $p = -0.02x + 400$. Find the profit made by selling the 3,001st phone each month.

    A. \$ 120

    B. \$ 40

    C. \$ 80

    D. -\$ 120

    E. -\$ 80

12. (3 points) If the demand function is given by $x^2p - 2p = 1$, what is the elasticity of demand if the price is \$ 40, and what would happen to the number of items sold if the price is increased?

    A. 0, the number of items sold will not change.

    B. 1, the number of items sold will not change.

    C. $\frac{3}{2}$. the number of items sold will decrease.

    D. $\frac{1}{162}$**, the number of items sold will increase also.**

    E. $\frac{3}{2}$, the number of items sold will increase also.

13. (3 points) Find $\frac{d^3y}{dx^3}$ of $y = \frac{2-x}{x}$

    A. $-\frac{x^4}{12}$

    B. $-\frac{12}{x^4}$

    C. $\frac{12}{x^4}$

    D. $-\frac{12}{x^3}$

    E. $-\frac{x^3}{12}$

14. (3 points) What are the only conditions that need to be met for a limit to exist?

    A. The left-sided limit has to exist

    B. The left-sided and right-sided limit both have to exist

    C. The function only needs to exist at that point

    D. The left-sided and right-sided limit both have to exist, and the function has to exist at that point.

    E. None of the above

15. (3 points) If the elasticity of demand is elastic, which one of the following is true?

   A. An increase in quantity results in no change in price.

   B. An increase in price results in a decrease in quantity.

   C. A decrease in price results in an increase in quantity.

   D. No matter the change in price, the quantity will not change.

   E. None of the above.

16. (3 points) Give the vertical asymptotes (if any) of the following function:

$$\frac{14x^2 + 37x + 24}{6x^2 - x - 15}$$

   A. $x = -\frac{3}{2}$ only

   B. $y = -\frac{3}{2}$ only

   C. $x = \frac{5}{3}$ **only**

   D. $x = -\frac{3}{2}$ and $x = \frac{5}{3}$

   E. There are no vertical asymptotes.

17. (3 points) Give the horizontal asymptotes (if any) of the following function:

$$f(x) = \frac{4x}{\sqrt{9x^2 - 23}}$$

   A. $x = \frac{4}{3}$

   B. $y = \frac{4}{3}$

   C. $y = \pm\frac{23}{9}$

   D. $y = \pm\frac{4}{3}$

   E. There are no horizontal asymptotes.

18. (3 points) Where is $f(x) = \frac{\frac{1}{2}x^2-4}{x+1}$ increasing and where is it decreasing?

A. Increasing from $(-\infty, -1)$ and from $(-1, \infty)$, decreasing nowhere

B. Decreasing from $(-\infty, -1)$, increasing from $(-1, \infty)$

C. Decreasing from $(-\infty, -4)$ and from $(4, \infty)$, increasing from $(-4, 4)$

D. Increasing from $(-\infty, \infty)$

E. Decreasing from $(-\infty, \infty)$

19. (3 points) Where are the relative minima and maxima (if any) of $f(x) = 2x^3 + \frac{13}{2}x^2 - 5x$?

A. $x = 0$ is an inflection point, and there are no minima or maxima

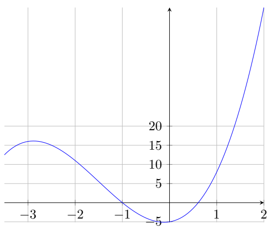
B. $x = \frac{1}{3}$ is a maximum, $x = -\frac{5}{2}$ is a minimum

C. $x = \frac{1}{3}$ is a minimum and $x = 0$ is a maximum

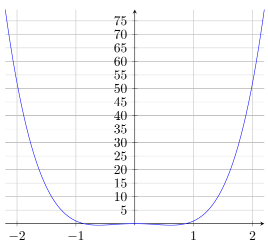
D. $x = 0$ is a minimum

E. $x = \frac{1}{3}$ is a minimum, $x = -\frac{5}{2}$ is a maximum

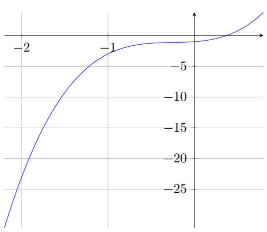
20. (3 points) Identify the graph of $f(x) = \frac{16}{3}x^3 + 9x^2 + 2x - 1$



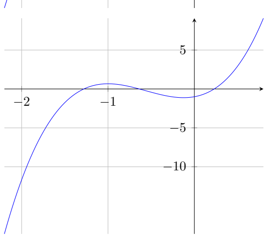
A.



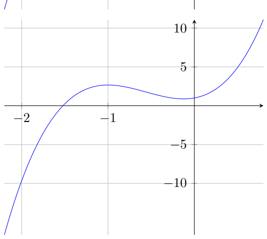
B.



C.



D.



E.

21. (3 points) James is building an enclosure to keep his horses in on the farm. The rectangular enclosure will need 3 fenced walls (the barn will provide the 4th wall), and he can buy 1600 feet of fencing to make the enclosure. What is the largest area he can enclose for his horses?

   A. 160,000 sq ft

   B. 640,000 sq ft

   C. 320,000 sq ft

   D. 80,000 sq ft

   E. None of the above

22. (3 points) Auburn alumni are doing a fundraiser selling T-Shirts for the football playoffs and have estimated that if they sell their T-shirts for $20, then 1,200 people will buy them, but for every $2 increase, 80 less people will buy them. They want to know the maximum revenue they can expect from this fundraiser. What is the maximum revenue under this model?

   (20+2x)(1200-80x) =

   A. $ 25,000

   B. $ 22,000

   C. $ 28,000

   D. $34,000

   E. None of the above

23. (3 points) What is the present value of $1,000,000 invested at 4% interest, compounded continuously for 30 years?

   A. $301,214.02

   B. $326,193,819.42

   C. $6.14

   D. $3,320,116.92

   E. $301,194.21

24. (3 points) Evaluate

$$\frac{dy}{dx} f(x) = \ln \frac{3x^5 e^x}{4x^2+1}$$

   A. $\frac{5}{x} + 1 - \frac{8x}{4x^2+1}$

   B. $\frac{5x^3 e^x}{8x+1}$

   C. $\frac{1}{3x^5} + \frac{1}{4x^2+1}$

   D. $\frac{x}{5} - \frac{2x}{2x+1}$

   E. $\frac{1}{x}$

25. (3 points) Evaluate

$$\int -4x^3 - \frac{3}{x} + \frac{2}{\sqrt{x^3}} dx$$

A. $-x^4 - \frac{3}{2x^2} - \frac{2}{\sqrt{\frac{1}{4}x^4}} + c$

B. $-x^4 - 3\ln|x| - \frac{4}{\sqrt{x}} + c$

C. $-x^4 + \frac{x}{x^2} - \frac{4}{5\sqrt{x^5}} + c$

D. $-x^4 - 3\ln|x| + \frac{4}{\sqrt{x}} + c$

E. $-x^4 - 3\ln|x| - \sqrt{x} + c$

26. (3 points) Evaluate

$$\int \frac{x}{1-4x^2} dx$$

A. $-4\frac{1}{1-4x^2} + c$

B. $-4\sqrt{(1-4x^2)} + c$

C. $-\frac{1}{8}\ln|1-4x^2| + c$

D. $-\frac{1}{4\sqrt{1-4x^2}} + c$

E. $\frac{1}{4}(\sqrt{1-4x^2})^3 + c$

27. (3 points) Evaluate

$$\int \frac{3y}{5y^2+4} dy$$

A. $\frac{3}{10}\ln|5y^2+4| + c$

B. $\frac{10}{3}\frac{1}{(5y^2+4)^2} + c$

C. $(5y^2+4)\ln|3y| + c$

D. $\frac{3}{10}(5y^2+4)\ln|3y| + c$

E. $\frac{3}{10} \cdot (10y)\ln|5y^2+4| + c$

28. (3 points) Evaluate

$$\int 2x^3\sqrt{x^2+1}dx$$

(Hint: It will require a second substitution. For it, use your original substitution equation.)

A. $\frac{2}{3}\sqrt{(x^2+1)^3} + \frac{2}{5}\sqrt{(x^2+1)^5} + c$

B. $-\frac{2}{5}\sqrt{(x^2+1)^5} - 2\sqrt{(x^2+1)^2} + c$

C. $\frac{2}{3}\sqrt{(x^2+1)^3} + c$

D. $x^2\sqrt{u} + c$

E. $\frac{2}{5}\sqrt{(x^2+1)^5} - \frac{2}{3}\sqrt{(x^2+1)^3} + c$

29. (3 points) Evaluate

$$\int_{-2}^{3} e^{-2x} dx$$

A. $\frac{1}{e^6} + 2e^4$

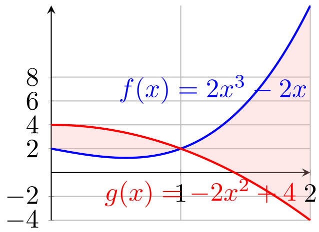
B. $e^6 - e^4$

C. $\frac{1}{e^6} - e^4$

D. $\frac{2}{e^6} - \frac{2}{e^4}$

E. $\frac{e^4}{2} - \frac{1}{2e^6}$

30. (3 points) Find the area bound by $f(x) = 2x^3 - 2x + 2$ and $g(x) = -2x^2 + 4$ on the interval [0,2]. (The curves intersect at (1,2)).



A. 8

B. 9

C. 10

D. 11

E. 12

31. (3 points) Find f_{yx} if $f(x,y) = \frac{x^2 y}{2y-1}$

A. $\frac{2xy}{2y-1}$

B. $\frac{2x}{(2y-1)^2}$

C. $\frac{2x(2y-1)-4xy}{(2y-1)^2}$

D. $-\frac{2x}{(2y-1)^2}$

E. $\frac{2xy}{2y-1}$

32. (3 points) Evaluate

$$\int_0^1 \int_0^1 3x^2y^4 + 2x - 4y dx dy$$

A. 0

B. $\frac{4}{5}$

C. $-\frac{4}{5}$

D. 1

E. $-\frac{2}{5}$

33. (3 points) What are the appropriate substitutions for

$$\int \frac{x}{x^2+1} + e^{x+e^x}dx \text{ ?}$$

(Hint: You'll need to remember/know that $e^{x+e^x} = e^x e^{e^x}$)

A. $u = x, v = e^x$

B. $u = x^2 + 1, v = e^x$

C. $u = x^2 + 1, v = x + e^x$

D. $u = x^2 + 1, v = x$

E. $u = x, v = x$

34. (3 points) If you were to calculate the volume of the solid encapsulated by the surface $f(x, y) = 4x^2 - 2xy + 8y^2$ and the region created by the parallelogram formed with the equations $x = 0, x = 2, y = x - 2,$ and $y = x + 4$, how would you set up the integral?

A. $\int_0^2 \int_{x-2}^{x+4} 4x^2 - 2xy + 8y^2 dydx$

B. $\int_0^2 \int_{x-2}^{y+4} 4x^2 - 2xy + 8y^2 dxdy$

C. $\int_{x-2}^{x+4} \int_0^2 4x^2 - 2xy + 8y^2 dydx$

D. $\int_0^2 \int_{x+4}^{x-2} 4x^2 - 2xy + 8y^2 dydx$

E. $\int_0^2 \int_{x+4}^{y-2} 4x^2 - 2xy + 8y^2 dxdy$