# Area Between Two Curves

Sometimes, it proves useful to us to be able to find the area between two curves, instead of between a curve and the x-axis.

### Area of the Region Between Two Curves

*Suppose $f$ and $g$ are two functions, and assume $f(x) > g(x)$ on an interval [a,b]. Then, the area bounded by the two curves $y = f(x)$ and $y = g(x)$ on [a,b] is given by*

$$\int_a^b [f(x) - g(x)]dx$$

*If, instead of an interval, [a,b], we wish to find the total area between two functions (regardless of interval), we have to find the intersection points of $f(x)$ and $g(x)$ to find our upper and lower limits of integration.*

Find the area of the region bounded by the function $f(x) = x^2$ and $g(x) = e^x + 1$ on the interval [0,2].

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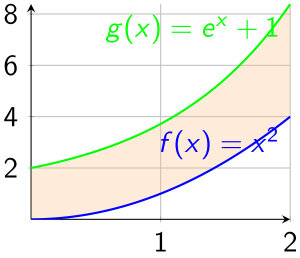

Find the area of the region bounded by the function $f(x) = x^2$ and $g(x) = e^x + 1$ on the interval [0,2].

$$\int_0^2 x^2 - (e^x + 1)dx \implies \int_0^2 x^2 - e^x - 1dx$$

$$\frac{x^3}{3} - e^x - x\Big|_0^2 \implies$$

$$\frac{2^3}{3} - e^2 - 2 - (\frac{0^3}{3} - e^0 - 0)$$

$$\frac{8}{3} - e^2 - 1 \approx 9.06$$

Find the area of the region bounded by the function $f(x) = x^2 - 2x - 1$ and $g(x) = -x^2 + 2x + 3$ on [0,2].

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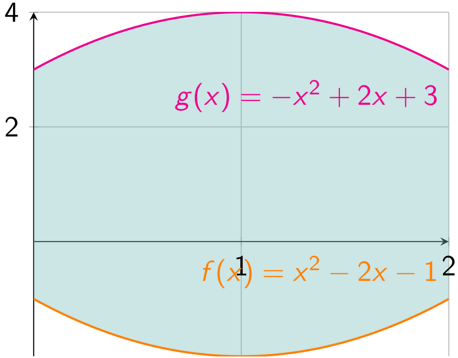

Find the area of the region bound by the function $f(x) = \frac{1}{x}$ and $g(x) = \sqrt{x}$ on the interval [1,4]

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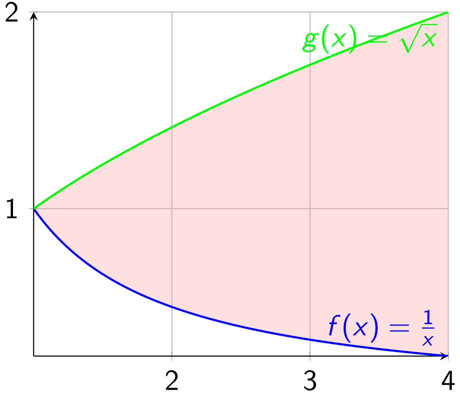
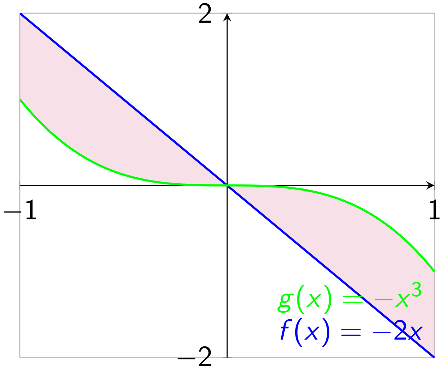
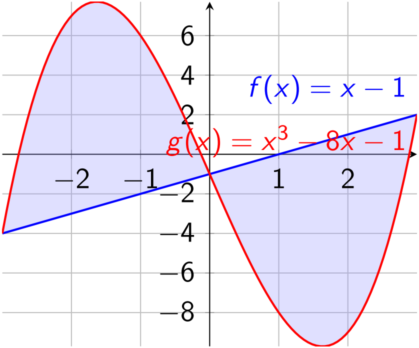


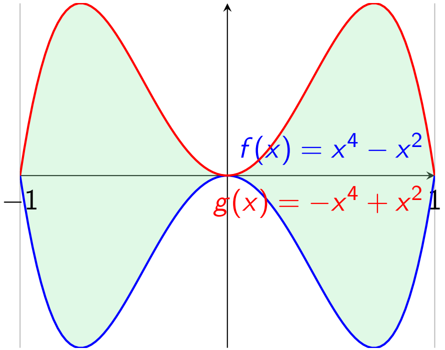
Find the area of the region bound by $f(x) = -2x$ and $g(x) = -x^3$ on the interval [-1,1]



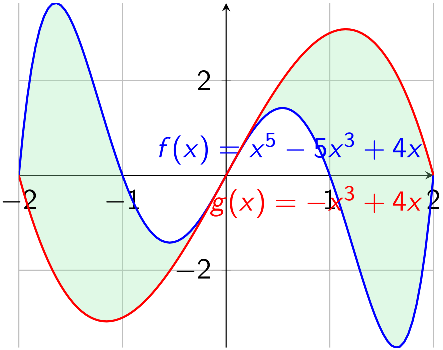
Find the total area of the region bound by the curves $f(x) = x - 1$ and $g(x) = x^3 - 8x - 1$



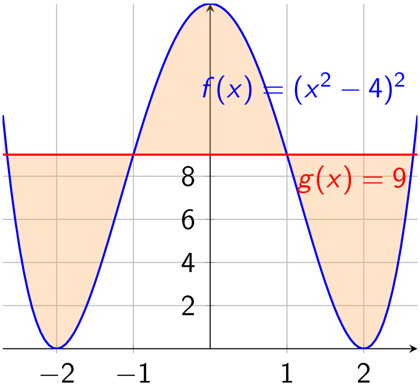
Find the total area of the region bound by the curves $f(x) = x^4 - x^2$ and $g(x) = -x^4 + x^2$



Find the total area of the region bound by the curves $f(x) = x^5 - 5x^3 + 4x$ and $g(x) = -x^3 - 4x$



Find the total area of the region bound by the curves $f(x) = (x^2 - 4)^2$ and $g(x) = 9$



Find the total area of the region bounded by the y-axis, $y = \sin x$, $y = \cos x$, and $x = \pi/2$.

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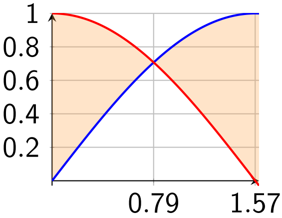

We could also have cases where we want to treat as functions of y, instead of functions of x.

### Area of the Region Between Two Curves

*Suppose $f$ and $g$ are two functions defined as a function of y. Then, the area bounded by the two curves $x = f(y)$ and $x = g(y)$ is given by*

$$\int_c^d [f(y) - g(y)]dy$$

*where $y = c$ and $y = d$ are the upper and lower boundaries.*

Find the area enclosed by the line $y = x - 1$ and the parabola $y^2 = 2x + 6$ (No picture given on this one!)

Find the area enclosed by the line $y = x - 1$ and the parabola $y^2 = 2x + 6$ (No picture given on this one!) Points of intersection are given by (-1,-2) and (5,4). As a function of y, these functions are:

$$x = \frac{1}{2}y^2 - 3 \text{ and } x = y + 1$$

Then, our integral is:

$$\int_{-2}^{4} [(y + 1) - (\frac{1}{2}y^2 - 3)]dy$$

Solving this yields an area of 18. Note we could've done this with respect to x also, but it would've been much more difficult.

## The Bottom Line

- *The most important thing when finding area between curves is knowing which function is on top (or to the right if a function of y) - be careful!*
- *If you have multiple regions, the function on top (or to the right, if a function of y) will probably change, so keep a close watch.*