Test 1 MATH1680 S/2018

Stuart Jones

Directions: *Below are 10 questions, each worth 10 points each. Some questions have multiple parts, and their point values are shown next to each part. Answer all questions completely. CIRCLE YOUR ANSWERS! Write only in the space provided. If more space is needed, use the back of each page.*

Question:	1	2	3	4	5	6	7	8	9	10	11	12	Total
Points:	10	10	10	10	10	10	10	10	10	10	0	0	100
Score:													

1. (10 points) Determine the length of $y = \ln(\sec x)$ between $0 \leq x \leq \pi/4$

   Solution: This is from the powerpoint. The answer is $\ln(\sqrt{2}+1)$

2. Find the limit of each sequence:

   (a) (5 points) $\{\frac{e^{2n}}{n}\}_{n=1}^{\infty}$

   Solution: This requires L'Hopital's Rule. The answer is ∞, ie, diverges.

   (b) (5 points) $\{\frac{(-1)^n}{n}\}_{n=1}^{\infty}$

   Solution: This requires the absolute value of a limit of a sequence theorem. The answer is 0.

3. (10 points) Review from Test 1: The line $y = x$ from $x = 0$ to $x = 1$ is rotated around the x-axis, forming a cone. Find the volume of this cone.

   Solution: This is the Disk Method from section 6.2, and is fairly straightforward. The answer is $\pi/3$.

4. (10 points) Determine if the following series converges or diverges:

$$\sum_{n=1}^{\infty} \frac{n^2+2}{n^4+5}$$

Solution: This can be solved by the Comparison Test with comparison series $\frac{n^2+2}{n^4}$, which splits into two series that are both convergent p-series. Since this comparison series is always larger than the original series, it forces the original series to converge.

5. (10 points) Determine the surface area of the solid formed by rotating $y = \sqrt{9 - x^2}, -2 \leq x \leq 2$ about the x-axis.

Solution: This is also from the powerpoint. The answer is 24π.

6. (10 points) Determine if the following series is convergent or divergent.

$$\sum_{n=0}^{\infty} ne^{-n^2}$$

Solution: This is an Integral Test problem. It takes a little bit of work to show that the function is decreasing (I'll be lenient on grading this if you don't show this.), but it is eventually decreasing, so the Integral Test applies. The integral converges to 1/2, so since the integral is convergent, the series must also be convergent by the Integral Test.

7. Find the sum of each series:
   (a) (5 points) $sum_n = 0^\infty 2^n 3^{1-n}$

   Solution: By the geometric series, the series converges to 9.

   (b) (5 points) $\sum_{n=2}^{\infty} \frac{1}{n^2-1}$

   Solution: This is a telescoping sum once partial fractions is used on the sequence. It converges to 3/4.

8. (10 points) Determine the center of mass of a semicircular plate of radius r centered at the origin.

Solution: This is from the book and powerpoint. The answer is $(0, \frac{4r}{3\pi})$

9. (10 points) A torus is formed by rotating a circle of radius r about a line in the plane of the circle that is a distance R (¿r) from the center of the circle. Find the volume of the torus.

Solution: This is from the book. The circle has area $A = \pi r^2$. By the symmetry principle, its centroid is its center, and so the distance traveled by the centroid during a rotation is $d = 2\pi R$. Therefore, by the Pappus Theorem, the volume of the torus is $V = Ad = (2\pi R)(\pi r^2) = 2\pi^2 r^2 R$

10. (10 points) Review from Test 2: Compute $\int \frac{1}{x^2-4} dx$

Solution: This is a simple partial fractions problem. The answer is $1/4(\ln(2-x) - \ln(x+2)) + c$

11. (5 points (bonus)) True/False: Must get all of these correct for points.

    1. If the series $\sum a_n$ diverges, then the limit of $a_n = 0$.
    2. Suppose f is a continuous, positive, decreasing function on $[1, \infty)$ and let $a_n = f(n)$. Then if $\sum a_n$ is divergent, then $\int_1^\infty f(x)dx$ is convergent.
    3. Suppose that we have two series $\sum a_n$ and $\sum b_n$ with $a_n > 0, b_n > 0$ for all n and $a_n \leq b_n$ for all n. Then, if a_n is convergent, then so is b_n.
    4. Suppose that we have two series $\sum a_n$ and $\sum b_n$ with both non-negative. If we have $c = \lim_{n\to\infty} \frac{a_n}{b_n}$ and if c is positive and finite, then both series converge to c.

Solution: These are all False.

12. (5 points (bonus)) Write 0.21212... as a fraction in lowest terms.

Solution: This can be written as a geometric series, and the answer is 7/33. (We did a similar one in class.)