

Section 3.6

Math 1000

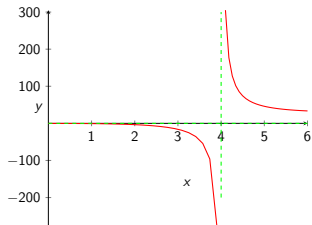
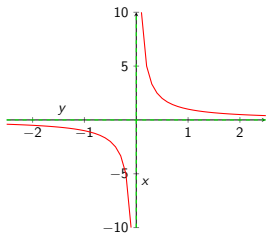
Stuart Jones

Rational Functions

Theorem

A function has a vertical asymptote at b if, as x approaches b from the left or the right, $y \rightarrow \pm\infty$. A function has a horizontal asymptote at b if, as x approaches $\pm\infty$, $y \rightarrow b$.

Examples



Theorem

The vertical asymptotes of a rational function are all areas where the denominator is 0.

If the degree of the numerator $>$ the degree of the denominator, there is no horizontal asymptote.

If the degree of the numerator $<$ the degree of the denominator, the horizontal asymptote is $y=0$.

If the degree of the numerator $=$ the degree of the denominator, the horizontal asymptote is $y = a/b$, where a is the leading numerator coefficient and b is the leading denominator coefficient.

Find all horizontal and vertical asymptotes (if any). $r(x) = \frac{1}{x-5}$

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Answer: Vertical $x= 5$, Horizontal $y=0$

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Answer: Vertical - $x = -2, x = 1/2$, Horizontal $y = 2$

Find all horizontal and vertical asymptotes (if any).

$$f(x) = \frac{8x^3 - 2}{2x^3 + 5x^2 + 7x}$$

Find all horizontal and vertical asymptotes (if any).

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Answer: Vertical $x = 0$, Horizontal $y = 4$

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Answer:

X-intercepts: $(-4,0)$, $(4,0)$

Y-intercepts: DNE

Sketch the graph of the function. To do so, find the intercepts and asymptotes first. $r(x) = \frac{8x-8}{x+2}$

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Intercepts

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Y-Intercept (0,-4)

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Intercepts

X-Intercept (1,0)

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Asymptotes

Vertical Asymptotes $x = -2$

Horizontal Asymptotes $y = 8$

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Intercepts

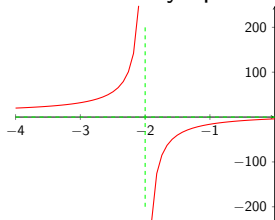
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Asymptotes

Vertical Asymptotes $x = 3$

Horizontal Asymptotes $y = 0$

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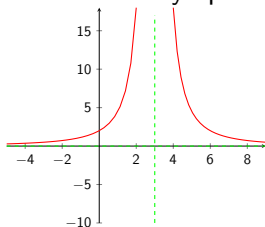
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Asymptotes

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Sketch the graph of the function. To do so, find the intercepts and asymptotes first. $r(x) = \frac{(x-1)(x+4)}{(x+1)(x-5)}$

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Asymptotes

Vertical Asymptotes $x = -1, x = 5$

Horizontal Asymptotes $y = 1$

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Intercepts

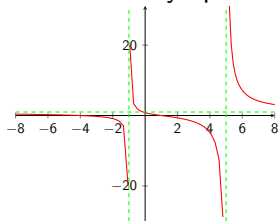
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Intercepts

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Y-Intercept (0,-1)

Asymptotes

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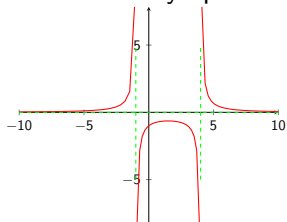
X-Intercept DNE

Y-Intercept (0,-1)

Asymptotes

Vertical Asymptotes $x = -1, x = 4$

Horizontal Asymptotes $y = 0$



The Bottom Line

- *Vertical asymptotes are found where the denominator is 0.*
- *Horizontal asymptotes are harder to find but you look at the leading terms of the numerator and denominator to do so.*
- *Once you have your intercepts and asymptotes, you can usually sketch a graph. This won't be exact, but it'll give you the idea of the graph.*